Scaffolding Instruction for All Students:

A Resource Guide for Mathematics

Grade 3

The University of the State of New York
State Education Department
Office of Curriculum and Instruction
and Office of Special Education
Albany, NY 12234

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Annmarie Urso, Ph.D., Associate Professor, State University of New York at Geneseo

Dee Berlinghoff, Ph.D., DB Consulting

Paul J. Riccomini, Ph.D., Associate Professor, Pennsylvania State University

Dawn Hamlin, Ph.D., Associate Professor, State University of New York at Oneonta

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Introduction

The Next Generation English Language Arts (ELA) and Mathematics Learning Standards intend to foster the 21st century skills needed for college and career readiness and to prepare students to become lifelong learners and thinkers. Learning standards provide the "destination" or expectation of what students should know and be able to do while teachers provide the "map" for getting there through high-quality instruction. **Lessons need to be designed to ensure accessibility to a general education curriculum designed around rigorous learning standards for all students, including students who learn differently (e.g., students with disabilities, English Language Learners (ELLs)/Multilingual Learners(MLLs), and other students who are struggling with the content).** It is vital that teachers utilize a variety of research-based instructional and learning strategies while structuring a student-centered learning environment that addresses individual learning styles, interests, and abilities present among the students in the class. Classrooms should be supportive and nurturing, and factors such as the age, academic development, English and home language proficiency, culture and background knowledge, and disability, should be considered when designing instruction. The principles of Universal Design for Learning should be incorporated into curricula to provide students with learning experiences that allow for multiple means of representation, multiple means of expression, and multiple means of engagement. These learning experiences will reduce learning barriers and foster equal learning opportunities for all students.

The purpose of this guide is to provide teachers with examples of scaffolds and strategies to supplement their instruction of ELA and mathematics curricula. Scaffolds are instructional supports teachers intentionally build into their lesson planning to provide students support that is "just right" and "just in time." Scaffolds do not differentiate lessons in such a way that students are working on or with different ELA texts or mathematical problems. Instead, scaffolds are put in place to allow all students access to grade-level content within a lesson. Scaffolds allow students to develop the knowledge, skills, and language needed to support their own performance in the future and are intended to be gradually removed as students independently master skills.

The scaffolds contained in this guide are grounded in the elements of explicit instruction as outlined by Archer and Hughes (2011). Explicit instruction is a structured, systematic approach to teaching which guides students through the learning process and toward independent mastery through the inclusion of clear statements regarding the purpose and rationale for learning the new skill/content; explanations and demonstrations of the instructional target; and supported practice with embedded, specific feedback.

The scaffolds in this guide can be adapted for use in any curricula and across content areas. While the exemplars were all drawn from the ELA and mathematics EngageNY modules, teachers are encouraged to customize the scaffolds in any lesson they deem appropriate. **All teachers (e.g., general, special education, English as a New Language, and Bilingual Education teachers) can use these scaffolds in any classroom setting to support student learning and to make the general education curriculum more accessible to all students without interfering with the rigor of the grade-level content.**

How to Use This Guide

The provision of scaffolds should be thoughtfully planned as to not isolate or identify any student or group of students as being "different" or requiring additional support. Therefore, in the spirit of inclusive and culturally responsive classrooms, the following is suggested:

- Make scaffolded worksheets or activities available to all students.
- Heterogeneously group students for group activities when appropriate.
- Provide ELLs/MLLs with opportunities to utilize their home language knowledge and skills in the context of the learning environment.
- Make individualized supports or adapted materials available without emphasizing the difference.
- Consistently and thoughtfully use technology to make materials more accessible to all students.

In the ELA guides, the *Table of Contents* is organized to allow teachers to access strategies based on the instructional focus (reading, writing, speaking and listening, and language) and includes a list of scaffolds that can be used to address those needs. In the mathematics guides, the *Table of Contents* is organized around the scaffolds themselves.

Each scaffold includes a description of what the scaffold is, who may benefit, and how it can be implemented in a lesson-specific model (see graphic below). **The *scripts* provided are only for demonstrating what a scaffold *might* look like in action.** Teachers are encouraged to make changes to presentation and language to best support the learning needs of their students. While lessons from the EngageNY modules are used to illustrate how each scaffold can be applied, the main purpose of the exemplars is to show how teachers can incorporate these scaffolds into their lessons as appropriate.

Title of Scaffold **Module: Unit: Lesson:**
Explanation of scaffold: This section provides a deeper explanation of the scaffold itself, including what it is and how it can and should be used. This section is helpful when implementing the scaffold in other lessons.
Teacher actions/instructions: This section provides specific instructions for the teacher regarding successful implementation of the scaffold.
Student actions: This section describes what the students are doing during the scaffolded portion of the lesson.
Student handouts/materials: This section indicates any student-facing materials that must be created to successfully use this scaffold.

Graphic Organizer (*RDW (Read, Draw, Write) Template*)

Exemplar from:
Module 1: Topic A: Lesson 1: Application Problem

Explanation of scaffold:
The *RDW Template* is a graphic organizer that can be used to support students who have difficulty organizing information and recalling multistep, problem-solving strategies. This template is intended to help students keep track of the steps involved in the RDW process that can be used to solve real world/application word problems. Some students may need additional scaffolding and explicit instruction to use this tool to structure their work to solve a mathematical problem. The following example shows one way to instruct students who need modeling and guided practice to use this tool and learn this new problem-solving strategy. Although the application problem in this lesson is used as an exemplar, graphic organizers such as the *RDW Template* can be used in any lesson to support students while learning a multistep problem-solving strategy without changing the rigor of the content.

Teacher actions/instructions:
Instruct students in the use of the RDW process and completion of the *RDW Template* to solve a mathematical problem as follows:

1. Read the problem.
2. Draw and label. Use a tape diagram, number bond, or array to make your drawing. Ask yourself, "What do I know? What do I need to find? How can I draw what I'm looking for?" Label your drawing.
3. Read the problem again.
4. Write an equation. Look at the evidence in your drawing, write an equation, and solve the problem.
5. Write a word sentence.

For students who require explicit instruction on how to use the RDW process to solve a mathematical problem and complete the *RDW Template*, the following sample *script* is provided to demonstrate one way instruction *might* look like:

Step 1: Read the problem.
T (teacher): *We are going to learn a strategy to help us solve problems. This strategy is called Read, Draw, Write, or RDW. We are going to use the RDW Template to gather the evidence, or important parts, from the word problem that we need to answer the question and solve the problem.*

Display a large version of the *RDW Template* on chart paper or use a document camera to project your work. Hand out student copies, and direct students to complete their *RDW Templates* to solve the problem as demonstrated.

Display the word problem:
There are 83 girls and 76 boys in third grade. How many total students are in third grade?

T: *The first step is* ***read****. That means I have to read the problem. What is step 1?*

S (student): *Read.*

T: *The problem says, "There are 83 girls and 76 boys in third grade. How many total students are in third grade?" I read the problem, so I can put a check in the box on my RDW Template.*

Step 2: Draw and label.
T: *Step 2 is* ***draw and label****. What is step 2?*

S: *Draw and label.*

T: *I am going to use a tape diagram to make my drawing. I need to ask myself, "What do I know?" I know from reading that the problem says there are 83 girls, so I am going to draw a tape diagram with 83. I also know the problem says there are 76 boys, so I am going to draw another tape diagram with 76. I must remember to write a label, so I don't forget what I'm drawing. I have to write "girls" next to the 83 and "boys" next to the 76.*

Step 3: Read the problem again.
T: *What is step 3?*

S: ***Read again.***

T: *Let's read the problem together.* [Chorally read the problem with students.] *We read the problem again, so we can put a check in the box.*

Step 4: Write an equation.
T: *What is Step 4?*

S: ***Write an equation.***

T: *We have to look at our drawings and see if we can use the evidence on my tape diagrams to write an equation. We see that third grade has 83 ___________.*

S: *Girls.*

T: *Look at your drawings again. How many boys are in third grade?*

S: *There are 76 boys.*

T: *That's correct. Now, the question is asking us to find out how many total students are in third grade. That means we have to find out how many boys and girls together make up third grade. Let's use **T** to represent the **total number of students** in third grade and label our diagrams. What do we need to do to find the total students in third grade?*

S: *We need to add the number of girls and the number of boys.*

T: *Right, again. Let's write and say the equation together.* [Write T = 83 + 76 on the *RDW Template* while chorally reading it aloud with students.] *What do we need to do next?*

S: *Solve the equation by adding together 83 and 76.*

T: *You got it! Do that now. If you need help, ask a person next to you.* [Write 83 + 76 = 159 on the *RDW Template*.] *We are almost finished!*

Step 5: Write a word sentence.

T: *What is step 5?*

S: ***Write a word sentence.***

T: *Finally, we have to write a sentence to answer the question. We have to remember to include all the information to tell the whole story. To tell how many total students are in third grade, we will write, "There are ____."*

S: *159 students in third grade.*

T: *Did we answer the question?*

S: *Yes!*

T: *Remember, we are going to use RDW and the RDW Template when we need to solve word problems.*

As students become more familiar with the process, fade the use of modeling and guided practice, and provide opportunities for students to work in pairs or small groups. Once students demonstrate the ability to use the RDW process with limited prompting, provide multiple, independent practice opportunities to ensure success.

Additional practice problems:

The garage has 98 cars and 42 motorcycles waiting for repair. How many total vehicles at the garage need to be fixed?

The bus took 52 students and 29 workers on its morning route. How many total people rode the bus this morning?

Our grocery store sells 41 gallons of chocolate ice cream and 87 gallons of vanilla ice cream every day. How many total gallons of chocolate and vanilla ice cream does our grocery store sell each day?

Student actions:

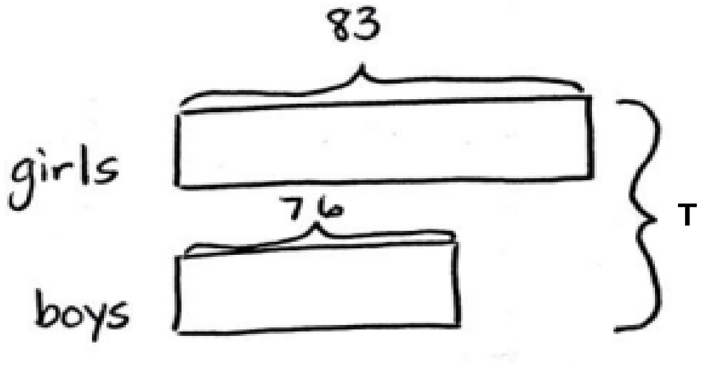
Students chorally respond and complete the *RDW Template*. Students may work in pairs or small groups to complete additional practice problems if needed.

Student handouts/materials:

RDW Template (found on page 6)

NAME: ______________________________

RDW Template (example)

Read	Make a ✔ after you read the problem. ☑
Draw and label	Draw a picture and label it. 
Read again	Make a ✔ after you read the problem again. ☑
Write	Write an equation. T=83+76 $\begin{array}{r} 83 \\ +76 \\ \hline 159 \end{array}$ T=159
Write	Write a sentence. There are 159 students in third grade.

NAME: ______________________________

RDW Template

Read	Make a ✔ after you read the problem. ☐
Draw and label	Draw a picture and label it.
Read again	Make a ✔ after you read the problem again. ☐
Write	Write an equation.
Write	Write a sentence.

Frayer Model

Exemplar from:
Module 1: Topic A: Lesson 3: Concept Development

Explanation of scaffold:
The Frayer model is a four-square graphic organizer that includes a student-friendly definition, a description of important characteristics, examples, and nonexamples. It provides a format to organize information and visual representations of the mathematical term being defined. Developing vocabulary skills is essential for students as they learn to *speak mathematically* and develop their abstract reasoning and problem-solving skills. The following example demonstrates how to provide explicit instruction for those students who need information broken down into smaller, more manageable chunks as well as modeling and guided practice to effectively use this tool to learn new vocabulary words. The term *factor* is used as an exemplar. However, the Frayer model can be use in any lesson to help students strengthen their conceptual knowledge and develop their understanding of unfamiliar vocabulary.

Teacher actions/instructions:
Select key mathematical terms. These terms should be limited in number and essential to developing a deeper understanding of the mathematical concepts or skills in the lesson.

Instruct students to complete Frayer models as follows:

1. Write the mathematical term in the middle circle.
2. Define the term, using student-friendly language, in the **Definition** box. Use your own words.
3. Write words to describe the term in the **Characteristics** box. Again, use your own words.
4. List examples of the definition in the **Examples** box. Draw a picture and/or write an equation to help you understand the term if needed.
5. List nonexamples of the definition in the **Nonexamples** box. Again, draw a picture and/or write an equation if needed.
6. Test yourself.

For students who require explicit instruction on how to use the Frayer model, the following sample *script* is provided to demonstrate one way instruction *might* look like :

Step 1: Write the mathematical term.
T (teacher): *We are going to use a graphic organizer called a Frayer model to help us understand what certain math terms, or vocabulary words, mean. It is very important we understand what a term means so we can understand what a math problem is asking us to find and can talk about math with others. Understanding vocabulary will make us better mathematicians!*

Display a large version of the Frayer model on chart paper, or use a document camera to project your work. Hand out student copies, and direct students to complete their Frayer models as demonstrated.

T: *We are going to learn about the word **factor.** What word?*

S (student): *Factor.*

T: *When we use the Frayer model, the first thing we do is write the vocabulary word in the middle circle. Let's write **factor** in the circle.*

Step 2: Define the term.

T: *You can see there are also 4 boxes. The first box is labeled **Definition**. A definition tells us the meaning of the term. **Factor** means a number that is multiplied by another number to get a product. Let's say that together.* [Chorally say the definition with students.] *Now, let's write that in the **Definition** box.*

Step 3: Describe the word in terms of its characteristics.

T: *The next box is **Characteristics**. This means we want to think of words and pictures and equations that describe **factor** or that are important to help us understand what it means.* [Draw two groups of three apples.] *If we wanted to use multiplication to find out how many total apples are here, what equation could we write?*

S: *2 x 3 = 6.*

T: *That's right. There are 2 groups of 3 apples, so we can write 2 x 3 = 6. In this equation, 2 is one **factor**. What is the other **factor**?*

S: *3.*

T: *Right again. We multiplied 2, the number of groups, and 3, the number of apples in each group, to find 6, the product, or total number of apples. The numbers 2 and 3 are **factors** of 6 because these are the numbers we multiplied to get the product. Factors in a multiplication problem can represent the number of groups or the size of the groups.* [Circle or highlight the factors, 2 and 3, and write down additional information as needed to describe **factor**.]

Step 4: List examples.

T: *The third box is **Examples**. Let's think of some more examples of **factors.** What are some **factors** of 8?*

S: *4 and 2.*

T: *Correct. 4 and 2 are **factors** of 8 because 4 x 2 = 8. Let's write down this equation and label each **factor**. Can you think of any other **factors** of 8?*

S: *1 and 8.*

T: *Why are 1 and 8 **factors** of 8?*

S: *Because 1 x 8 = 8.*

T: *You got it! Let's write down this equation in the **Examples** box also and label each **factor**.*

Step 5: List nonexamples.

T: *The last box is **Nonexamples**. This is an important box because it shows we really understand what the word means and what it doesn't mean. We've already written down all the examples of **factors** of 8. Now, let's think of some nonexamples. Name a number that is <u>not</u> a factor of 8.*

S: *3.*

T: *Why isn't 3 a **factor** of 8?*

S: *Because there's not a number you can multiply 3 by to get 8.*

T: *Excellent! You remembered that a **factor** is a number that we multiply by another number to get a product. Since we can't multiply 3 by any <u>whole</u> number to get the product 8, 3 is <u>not</u> a **factor** of 8. Can you think of any other nonexamples of a **factor** of 8?* [Write down any reasonable answers.]

Step 6: Test yourself.

The study step is critical to student success in using vocabulary strategies such as the Frayer model. Students need to study the terms to internalize them for later use. Students can quiz each other during "down times," or the models/cards can be used as part of a center activity.

Instruct students to study their Frayer models as follows:

1. Cover each box of the Frayer model with a sticky note. Do not cover the math term in the middle circle.
2. Say the term in the middle and try to say the definition.
3. If you do not know the definition, uncover the **Characteristics** box, and try to say the definition.
4. If you do not know the definition, uncover the **Examples** box, and try to say the definition.
5. If you do not know the definition, uncover the **Nonexamples** box, and try to say the definition.
6. If you do not know the definition, uncover the **Definition** box.

Repeat steps 1-6 for each Frayer model.

<u>Student actions</u>:
Students work either individually or in pairs to make and study Frayer models.

<u>Student handouts/materials</u>:
Frayer Model template (found on page 11)
Sticky notes

NAME: ____

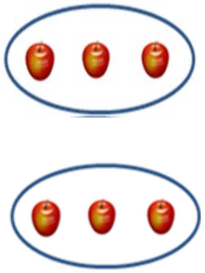
Frayer Model (example)

Factor

Definition

A number that is multiplied by another number to get a product.

Characteristics



2 groups of 3 apples

The unknown factor could represent the size of the group or the number of groups.

2 x $\underline{\ 3\ }$ = 6 **3 is how many in each group.**

$\underline{\ 2\ }$ x 3 = 6 **2 is the number of groups.**

Examples

$\underset{\text{factor}}{\underset{\nearrow}{4}} \times \underset{\text{factor}}{\underset{\nwarrow}{2}} = 8$ $\underset{\text{factor}}{\underset{\nearrow}{1}} \times \underset{\text{factor}}{\underset{\nwarrow}{8}} = 8$

1, 2, 4, and 8 are **factors** of 8.

Nonexamples

3, 5, 6, and 7 are **not** factors of 8.

NAME: ______________________________________

Frayer Model

Definition	**Characteristics**
Examples	**Nonexamples**

Desk Reference Sheet (*How to Round Numbers*)

Exemplar from:
Module 2: Topic C: Lesson 12: Concept Development

Explanation of scaffold:
The *How to Round Numbers* desk reference sheet supports students who have difficulty understanding the concept of rounding and recalling the multistep process involved. An explicit rule for rounding numbers provides additional support to students who need to develop a concrete understanding of this mathematical process which is important to the content of the lesson and will be necessary to use in the future. Although the *How to Round Numbers* desk reference sheet connects to and uses the concept development section in this lesson as an exemplar, desk reference sheets can be used as a visual support in any lesson until students build fluency remembering key mathematical terms, concepts, processes, and/or skills.

Teacher actions/instructions:
Instruct students in the process for rounding numbers and use of the *How to Round Numbers* desk reference sheet as follows:

1. Find and label the benchmarks/endpoints.
2. Find and label the midpoint/halfway point.
3. Plot and label the number you need to round.
4. Round using the rule: *If the number is below the midpoint/halfway point, then round down. If the number is at the midpoint/halfway point or above, then round up.*

For students who require explicit instruction on how to use the *How to Round Numbers* desk reference sheet to round a number, the following sample *script* (based on the language found in the concept development section of Module 2, Topic C, Lesson 12) is provided to demonstrate one way instruction *might* look like:

Step 1: Find and label the benchmarks/endpoints.
T (teacher): *We are going to learn to round two-digit numbers to the nearest ten. We are going to use a vertical number line to help us do that. A vertical number line goes up and down instead of across, or horizontal. Objects such as thermometers and measuring cups use vertical number lines.*

Display a large version of the *How to Round Numbers* desk reference sheet on chart paper, or use a document camera to project your work. Hand out student copies (these should be placed in plastic sleeves, so students can write on them and erase during lessons), and direct students to use their number lines to round numbers as demonstrated.

T: *This beaker has 73 milliliters of water in it.* [Show a beaker holding 73 milliliters of water.] *We want to round the number 73 to the nearest ten. The first thing we need to do is find and label the benchmarks on our number line. How many tens are in 73?*

S (student): *7 tens.*

T: *That's right. Follow along with me on the number line on your How to Round Numbers sheet.* [Write **70 = 7 tens** to the right of the lowest benchmark.] *What is 1 more ten than 7 tens?*

S: *8 tens.*

T: *Right again.* [Write **80 = 8 tens** to the right of the highest benchmark.]

Step 2: Find and label the midpoint.

T: *The second thing we need to do is find and label the midpoint on our number line. A midpoint is a point that is exactly in the middle between two points. This point will be used to help us see where 73 falls with respect to the 7 tens (70) or the 8 tens (80). What number is halfway between 7 tens and 8 tens?*

S: *75.*

T: *Correct. The number 75 is halfway between 7 tens and 8 tens. 75 = 7 tens and 5 ______.*

S: *Ones.*

T: *Let's write 75 = 7 tens 5 ones to the right of the tick mark showing the mid- or halfway point on the number line.*

Step 3: Plot and label the number you need to round.

T: *The next step is to plot 73 on our number line. Remind me, what unit of measurement are we plotting on our number line?*

S: *Milliliters.*

T: *You got it. Say, "Stop!" when my finger points to where 73 milliliters should be.* [Move your finger up the number line from 70 toward 75.]

S: *Stop!*

T: *Now, let's put a tick mark on the number line where you said to stop. To label, we will write*
73 = 7 tens _______ _______.

S: *7 tens 3 ones.*

T: *Excellent. Write the label to the right of the tick mark you drew on your number line now.*

Step 4: Round using the rule.

T: *Now that we know where 73 milliliters is, we can round the measurement to the nearest 10 milliliters. Look at your vertical number line. Is 73 milliliters more than halfway or less than halfway between 70 milliliters and 80 milliliters?*

S: *Less than halfway.*

T: *We know 73 milliliters is less than halfway between 70 milliliters and 80 milliliters because 3 is less than 5, and 5 marks the mid- or halfway point. Looking at your number line, what is 73 milliliters rounded to the nearest ten. Remember to use the rule:* ***If the number is below the midpoint/halfway point, then round down. If the number is at the midpoint/halfway point or above, then round up.***

S: *73 milliliters rounded to the nearest ten is 70 milliliters.*

T: *Correct. Another way to say it is that 73 milliliters is about 70 milliliters.*

As students become more familiar with the process, fade the use of modeling and guided practice, and provide opportunities for students to work in pairs or small groups. Once students demonstrate the ability to round numbers with limited prompting, provide multiple, independent practice opportunities to ensure success. Explain that the vertical number line can be used even when the units of measurement change.

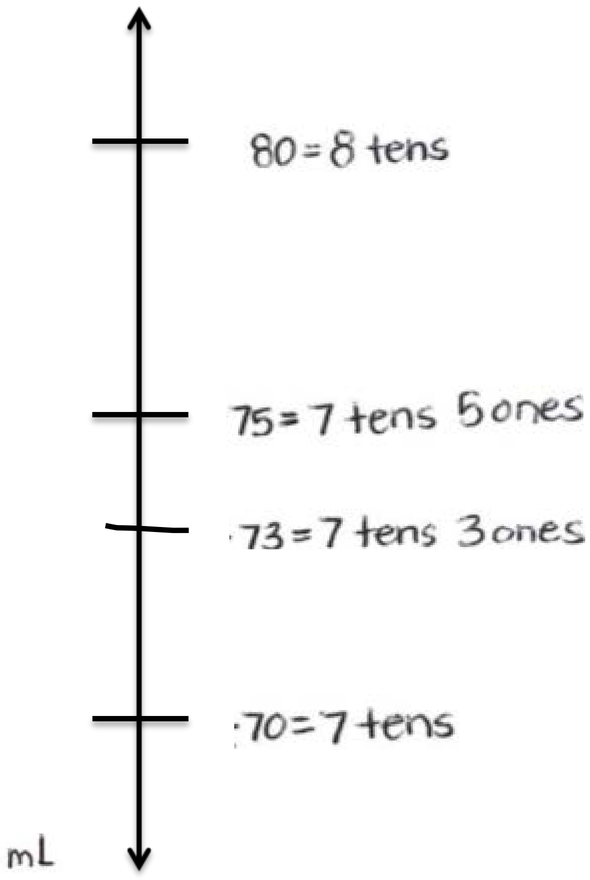
Additional practice for rounding to the nearest ten:

61 centimeters
38 minutes
25 grams

Student actions:
Students chorally respond and use the *How to Round Numbers* desk reference sheet to follow along with the teacher's model. Students then work in pairs, small groups, or independently as appropriate to round numbers using the *How to Round Numbers* desk reference sheet as needed.

Student handouts/materials:
How to Round Numbers desk reference sheet (found on the page 16)

How to Round Numbers (example)



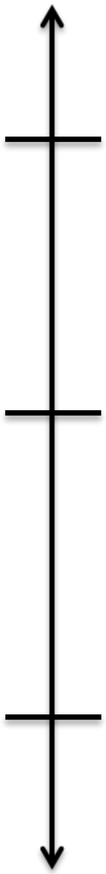
1. Find and label the benchmarks/endpoints.

2. Find and label the midpoint/halfway point.

3. Label the number you need to round.

4. Round using the rule: *If the number is **below the midpoint/halfway point**, then **round down**. If the number is **at the midpoint/halfway point or above**, then **round up**.*

How to Round Numbers



1. Find and label the benchmarks/endpoints.

2. Find and label the midpoint/halfway point.

3. Label the number you need to round.

4. Round using the rule: *If the number is **below the midpoint/halfway point**, then **round down**. If the number is **at the midpoint/halfway point or above**, then **round up**.*

Concrete-Representational-Abstract (CRA)

Exemplar from:
Module 3: Topic A: Lesson 2: Concept Development

Explanation of scaffold:
CRA is a three-part instructional strategy in which the teacher begins by modeling and thinking aloud with **concrete** objects (e.g., blocks, disks, etc.), and then progresses to **representing** the concrete objects with drawings. The final level is the **abstract** level, where only numbers and mathematical symbols are used to complete the algorithm. Although the following exemplar connects to and uses the concept development section in this lesson as an exemplar, CRA is a method that can be used in any lesson when teaching abstract concepts that are difficult for students to understand.

The Concept Development for this lesson centers on expanding students' study of factors, specifically working with factors of seven. CRA provides additional support for students to see the groups of five that are within a factor of seven.

Teacher actions/instructions:
In this module lesson, it is suggested that students requiring additional concrete support start out by using cubes or a Rekenrek. Provide student partners with linking cubes of two different colors or a Rekenrek to allow opportunities for guided practice with concrete materials before transitioning to the representational/pictorial, and eventually to the abstract examples. Instruct students to share the linking cubes or Rekenrek with their partners, work together, and follow along as you model how to use these manipulatives to solve the problem. Use a document camera to project your work.

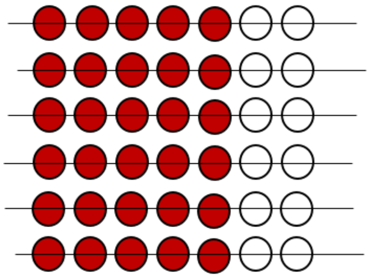
For students who require explicit instruction on how to use the materials provided to use familiar five facts to solve multiplication problems and apply the commutative and distributive properties, the following sample script is provided to demonstrate one way instruction might look like:

Concrete

T (teacher): *Today, we will be using familiar multiplication facts to help us discover new ones. We will be using the commutative and distributive properties as well. Our first problem is to determine what 6 x 7 is. What does 6 x 7 mean? It means there are 6 groups of __________.*

S (student): *Seven.*

T: *That's right. I'm going to use my Rekenrek to show 6 groups of seven, or the multiplication fact, 6 x 7. Please work with your partner to do the same.* [If a Rekenrek is unavailable, students can use linking cubes to show the six groups of seven, highlighting groups of five within the seven.]



T: *If I look at the rows of my Rekenrek, I see 6 groups of seven. If I count all of the beads row by row, I would get 42. So, 6 x 7 must be equal to what?*

S: *42.*

T: *That is correct. Now, I wonder if there is an easier way to get to 42 without counting all the beads. Remember how I said that we are going to use familiar multiplication facts to help us discover new ones? Well, let's look at our Rekenrek and see if it highlights special groups within the rows that can help us get to the same answer. What other sized groups do you see in each row?*

S: *Groups of 5 and groups of 2.*

T: *Yes, let's start with the groups of 5. How many groups of 5 do you see? (Groups of red beads)*

S: *Six.*

T: *Yes. So how many total red beads are there in the 6 groups of 5? Use your Rekenrek and skip count by fives if you need to. Turn to your partner and say the multiplication fact for 6 fives. What is the multiplication fact for the 6 fives and how many total red beads are there in those 6 fives?*

S: *6 x 5 =30.*

T: *You are correct, there are 30 total red beads. Now do the same for the groups of two. How many groups of two do you see? (Groups of white beads)*

S: *Six.*

T: *Yes. So how many total white beads are there in 6 groups of 2? Use your Rekenrek and skip count by twos if you need to. Turn to your partner and say the multiplication fact for 6 twos. What is the multiplication fact for the 6 twos and how many total white beads are in those 6 twos?*

S: *6 x 2 =12.*

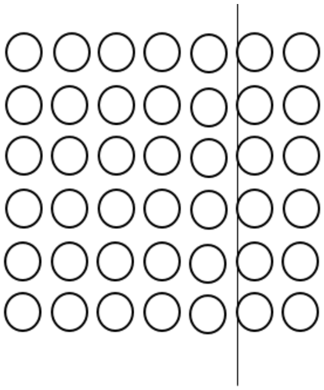
T: *Great, now what do you think I need to do with the 30 (red beads) and the 12 (white beads) to get the total number of beads that make up the 6 groups of 7?*

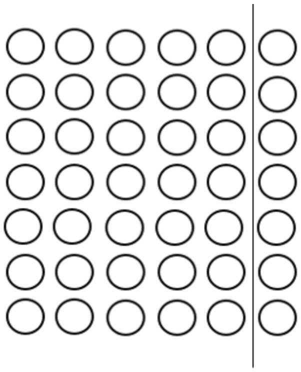
S: *Add them together.*

T: *That is correct, 6 groups of 7 is the same as 6 groups of 5 plus 6 groups of 2, 30 + 12, which is 42. 6 groups of 7=6 groups of 5 +6 groups of 2.*

Representational

Students can be asked to draw an array that represents 6 x 7. Diagrams will look like the concrete models. Students may draw an array that highlights 6 groups of 7, or they may draw an array that highlights 7 groups of 6. Based on previous work with the commutative property, students should understand that both represent the same product of 42.





T: *Let's look at the first array. It shows 6 groups of 7. I would need to count by sevens, six times, to get my answer. Maybe, sevens are too big. Suppose I look for a familiar group within the group of seven that I can count by. I like groups of fives. I will draw a line to highlight the five groups that are in each row. When I break a seven into a group of five, I am also forming a group of how many?*

S: *A group of two.*

T: *That is correct. I broke each group of seven into a group of five and a group of two. Let's start with the groups of five. How many groups of five do I have?*

S: *Six.*

T: *That is correct. So how many circles are there total in the 6 groups of five?* (Students can skip count if necessary. They can also color in the groups of 5 as well.)

S: *30.*

T: *That is correct, 6 groups of five, or 6 x 5 =30. Now, what about the groups of two. How many groups of two do I have?*

S: *Six.*

T: *That is correct. So how many circles are there total in the 6 groups of two?* (Students can skip count by two if necessary.)

S: *12.*

T: *That is correct, 6 groups of two, or 6 x 2=12. So how can I use the numbers 30 and 12 to find out what 6 x 7 is equal to?*

S: *We can add them together.*

T: *Exactly, 6 groups of seven is equal to the 6 groups of five plus the 6 groups of two. 30 +12 =42, so 6 x 7 must equal 42.*

T: *Now, let's look at the second array. It shows 7 groups of 6 (7 x 6), which I know from the commutative property is the same as 6 x 7. In this array, I would need to count by sixes, seven times, to get my answer. Maybe, sixes are challenging to count by. Suppose I look for a familiar group within the group of six that I can count by. I like groups of fives. I will draw a line to highlight the five groups that are in each row. But when I break a six into a group of five, I am also forming a group of how many?*

S: *One.*

T: *That is correct. I broke each group of six into a group of five and a group of one. Let's start with the groups of five. How many groups of five do I have?*

S: *Seven.*

T: *That is correct. So how many circles are there total in the 7 groups of five?* (Students can skip count if necessary and color in the groups of five.)

S: *35.*

T: *That is correct, 7 groups of five, or 7 x 5 =35. Now, what about the groups of one. How many groups of one do I have?*

S: *Seven.*

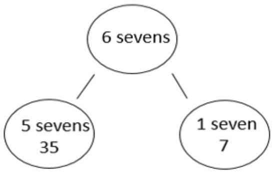
T: *That is correct. So how many circles are there total in the 7 groups of one?* (Students can count if necessary.)

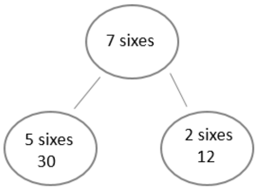
S: *7.*

T: *That is correct, 7 groups of one, or 7 x 1=7. So how can I use the numbers 35 and 7 to find out what 7 x 6 is equal to?*
S: *We can add them together.* (35 + 7 is the same as 35 + 5 +2)

T: *Exactly, 7 groups of six is equal to the 7 groups of five plus the 7 groups of one. 35 + 7, so 7 x 6 must equal 42.*

Students may choose to use other pictorial representations, such as number bonds to represent the problem.
(6 x 7=7 x 6):





Abstract

T: *We need to be able to write an equation that shows how we break apart a multiplication fact into two easier facts, using our five facts. This is very similar to how we used our Rekenrek and the arrays. Let's look at 6 x 7. 6 x 7 means 6 groups of seven. If I break the seven into a group of five, what size group is left?*

S: *A group of two.*

T: *That is correct. So, one group of seven is the same as one group of five plus one group of two. We now need six groups of seven. So, six groups of seven would be the same as six groups of five plus six groups of two. How do we write six groups of five as a multiplication fact?*

S: *6 x 5.*

T: *Excellent. How do we write six groups of two as a multiplication fact?*

S: *6 x 2.*

T: *Right again. So now we can take the fact that we know six groups of seven is the same as six groups of five plus six groups of two and translate it into an equation*. (Write the statement on board, with the equation form underneath.)

Six groups of seven is the same as six groups of five plus six groups of two.
6 x 7 = (6 x 5) +(6 x 2)
6 x 7 = 30 + 12
6 x 7 = 42

Following from the second array, a similar approach can be used to translate the breaking down of 7 x 6 into two easier facts, using five facts.

Seven groups of six is the same as seven groups of five plus seven groups of one.
7 x 6= (7 x 5) + (7 x 1)
7 x 6 = 35 + 7
7 x 6 = 42

Provide additional opportunities for students to reinforce their understanding of the commutative and distributive properties while using their familiar multiplication five facts to complete problems using concrete, representational, and/or abstract means. As students become more familiar with the strategy, fade the use of modeling and guided practice.

Additional problems from the module lesson:
6 x 9
One group of 9 is the same as a group of five plus a group of ______.
Six groups of 9 is the same as six groups of five plus six groups of _____.
6 x 9 = (6 x 5) + (   )

9 x 6
One group of 6 is the same as a group of five plus a group of ______.
Nine groups of 6 is the same as nine groups of five plus nine groups of ______.
9 x 6 = (9 x 5) + (   )

6 x 6 = ?

Student actions:
Students chorally respond, arrange Rekenrek or linking cubes, and work in pairs to complete problems.

Student handouts/materials:
Rekenrek or linking cubes (two different colors)
Personal white boards
*****Note: Students may choose to use other pictorial representations, such as number bonds, to represent problems and distribute differently depending on their use of the commutative property.**

Worked Problems

Exemplar from:
Module 7: Topic A: Lesson 2: Homework

Explanation of scaffold:
Worked problems provide support as students go through the learning stages of acquisition to proficiency to fluency to generalization and can be used to build learners' momentum and self-efficacy. Students are provided models of completed and/or partially completed problems while they work on developing and applying a newly learned skill without corrective feedback from the teacher.

The use of worked problems is based on the *Interleave Worked Solution Strategy* (*IWSS*) and involves alternating between fully worked and/or partially completed examples that students can use as a reference and practice problems for students to complete independently. A high level of scaffolding can be provided by giving worked examples and practice problems that are very similar in structure and by providing annotations or problem-solving steps alongside worked examples. Support can be faded by providing fewer worked examples or providing practice problems that are less similar to worked examples. Although the homework problems in this lesson are used as an exemplar, worked problems can be used in any lesson to support students who understand the mathematical concept involved but need examples as they practice what they have learned during the day's lesson to complete homework assignments.

Teacher actions/instructions:
Add worked problems to homework sheets. Provide the adapted sheets as needed to students, and direct them to complete the assigned problems. Instruct students to fill in the information needed to solve partially completed problems wherever they see a blank line. You may consider additional scaffolding by assigning specific problems on a homework sheet for those students who are likely to benefit from a shorter period of practice in which they are able to complete the task, rather than a longer period of practice in which they are unsuccessful.

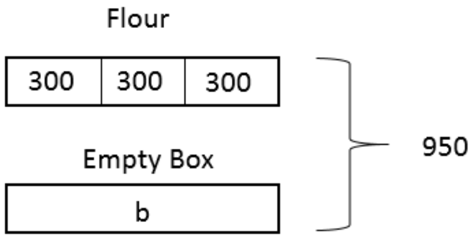
Student actions:
Students complete problems on the adapted sheets as assigned.

Student handouts/materials:
Lesson 2 Homework sheets (found on the following pages)
*****Note: The module lesson homework sheet included word problems 1-6 only. Supporting information, including tape diagrams, was added to scaffold student learning.**

Name ____ **Date** ________________________

Use the RDW process to solve. Use a letter to represent the unknown in each problem.

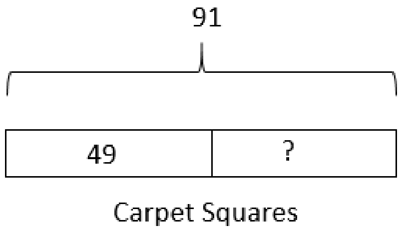
1. A box containing 3 small bags of flour weighs 950 grams. Each bag of flour weighs 300 grams. How much does the empty box weigh?



3 x 300=900
b=950-900
b = 50

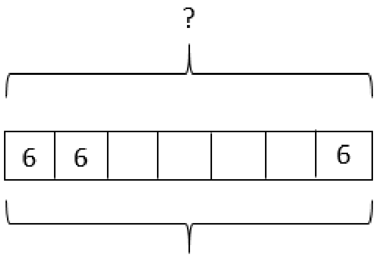
The box holding the 3 small bags of flour weighs 50 grams.

2. Mr. Cullen needs 91 carpet squares. He has 49 carpet squares. If the squares are sold in boxes of 6, how many more boxes of carpet squares does Mr. Cullen need to buy?



91
- 49
?

? is the number of **carpet squares** Mr. Cullen needs to buy.

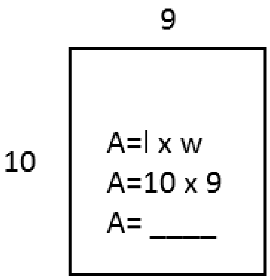


The number of **boxes** of 6.

b is the number of **boxes** of 6 that Mr. Cullen needs to buy
b = _____
b = ? ÷ 6

Mr. Cullen needs to buy _____ **boxes** of carpet squares.

3. Erica makes a banner using 4 sheets of paper. Each paper measures 9 inches by 10 inches. What is the total area of Erica's banner?



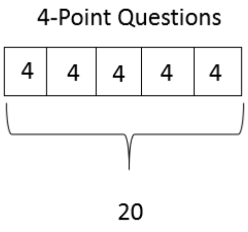
A=l x w
A=10 x 9
A= _____

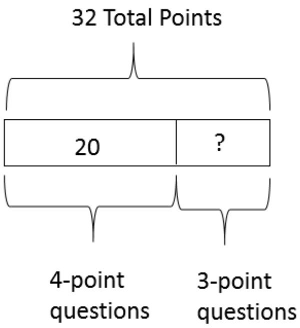
Each sheet of paper has an area of ___ square inches.

___ square inches	___ square inches
___ square inches	___ square inches

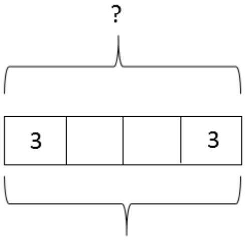
The total area of Erica's banner is _____ square inches.

4. Monica scored 32 points for her team at the Science Bowl. She got 5 four-point questions correct, and the rest of her points came from answering three-point questions. How many three-point questions did she get correct?





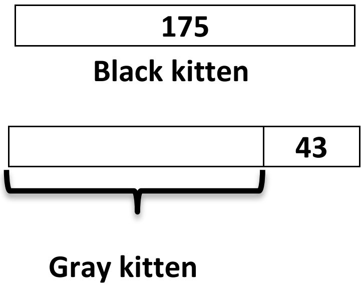
? is the total amount of points for the 3-point questions.

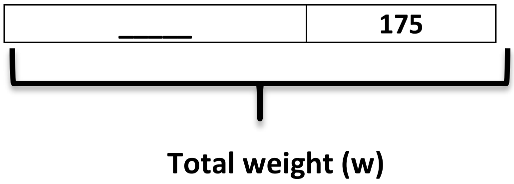


The number of 3-point questions that Monica got correct.

q is the number of 3-point questions Monica got correct.
q=______
q=? ÷3
Monica got _____ 3-point questions correct.

5. Kim’s black kitten weighs 175 grams. Her gray kitten weighs 43 grams less than the black kitten. What is the total weight of the two kittens?





175 - 43 = _____

w = 175 + _____

w = _____

The total weight of the two kittens is ______________________.

6. Cassias and Javier’s combined height is 267 centimeters. Cassias is 128 centimeters tall. How much taller is Javier than Cassias?

References

Archer, A. and Hughes, C. (2011). *Explicit instruction: Effective and efficient teaching.* New York, NY: The Guilford Press.